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Estimating a mean matrix: boosting efficiency by multiple affine shrinkage

Rudolf Beran

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Abstract The unknown matrix M is the mean of the observed response matrix in a multivariate linear model with independent random errors. This paper constructs regularized estimators of M that dominate, in asymptotic risk, least squares fits to the model and to specified nested submodels. In the first construction, the response matrix is expressed as the sum of orthogonal components determined by the submodels; each component is replaced by an adaptive total least squares fit of possibly lower rank; and these fits are then summed. The second, lower risk, construction differs only in the second step: each orthogonal component is replaced by a modified Efron-Morris fit before summation. Singular value decompositions yield computable formulae for the estimators and their asymptotic and estimated risks. In the asymptotics, the row dimension of M tends to infinity while the column dimension remains fixed. Convergences are uniform when signal-to-noise ratio is bounded.

Keywords Total least squares · Efron-Morris fit · Penalized least squares · Regularization · Rank reduction