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Local empirical processes near boundaries of convex bodies

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Abstract We investigate the behaviour of Poisson point processes in the neighbourhood of the boundary ∂K of a convex body K in $\mathbb{R}^d, d \geq 2$. Making use of the geometry of K, we show various limit results as the intensity of the Poisson process increases and the neighbourhood shrinks to ∂K. As we shall see, the limit processes live on a cylinder generated by the normal bundle of K and have intensity measures expressed in terms of the support measures of K. We apply our limit results to a spatial version of the classical change-point problem, in which random point patterns are considered which have different distributions inside and outside a fixed, but unknown convex body K.

Keywords Poisson point process · Convex body · Empirical process · Support measure · Normal cylinder · Change-set problem · Limit process