

## Limit laws for the Randić index of random binary tree models

Qunqiang Feng · Hosam M. Mahmoud ·  
Alois Panholzer

Received: 20 February 2006 / Revised: 18 July 2006 / Published online: 10 February 2007  
© The Institute of Statistical Mathematics, Tokyo 2007

**Abstract** We investigate the Randić index of random binary trees under two standard probability models: the one induced by random permutations and the Catalan (uniform). In both cases the mean and variance are computed by recurrence methods and shown to be asymptotically linear in the size of the tree. The recursive nature of binary search trees lends itself in a natural way to application of the contraction method, by which a limit distribution (for a suitably normalized version of the index) is shown to be Gaussian. The Randić index (suitably normalized) is also shown to be normally distributed in binary Catalan trees, but the methodology we use for this derivation is singularity analysis of formal generating functions.

**Keywords** Random trees · Binary search trees · Catalan trees · Recurrence · Moments · Contraction method · Functional equation · Computational chemistry · Chemical index · Topological index