## Saddlepoint approximations for multivariate *M*-estimates with applications to bootstrap accuracy

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**Abstract** We obtain marginal tail area approximations for the one-dimensional test statistic based on the appropriate component of the *M*-estimate for both standardized and Studentized versions which are needed for tests and confidence intervals. The result is proved under conditions which allow the application to finite sample situations such as the bootstrap and involves a careful discretization with saddlepoints being used for each neighbourhood. These results are used to obtain second-order relative error results on the accuracy of the Studentized and the tilted bootstrap. The tail area approximations are applied to a Poisson regression model and shown to have very good accuracy.

**Keywords** Empirical saddlepoint  $\cdot$  Relative errors  $\cdot$  Studentized *M*-estimates  $\cdot$  Tail area approximation  $\cdot$  Tilted bootstrap