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Estimating a general function of a quadratic function

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Abstract Let $x \in \mathbb{R}^p$ be an observation from a spherically symmetric distribution with unknown location parameter $\theta \in \mathbb{R}^p$. For a general non-negative function c, we consider the problem of estimating $c(\|x-\theta\|^2)$ under the usual quadratic loss. For $p \geq 5$, we give sufficient conditions for improving on the unbiased estimator γ_0 of $c(\|x-\theta\|^2)$ by competing estimators $\gamma_s = \gamma_0 + s$ correcting γ_0 with a suitable function s. The main condition relies on a partial differential inequality of the form $k\,\Delta s + s^2 \leq 0$ for a certain constant $k \neq 0$. Our approach unifies, in particular, the two problems of quadratic loss estimation and confidence statement estimation and allows to derive new results for these two specific cases. Note that we formally establish our domination results (that is, with no recourse to simulation).

Keywords Loss estimation · Confidence statement · Spherically symmetric distribution · Green integral formulas · Sobolev spaces · Differential inequations