Yoshio Takane · Yongge Tian · Haruo Yanai

On constrained generalized inverses of matrices and their properties

Received: 24 October 2005 / Revised: 28 March 2006 / Published online: 8 August 2006 © The Institute of Statistical Mathematics, Tokyo 2006

Abstract A matrix *G* is called a generalized inverse (*g*-inverse) of matrix *A* if AGA = A and is denoted by $G = A^-$. Constrained *g*-inverses of *A* are defined through some matrix expressions like $E(AE)^-$, $(FA)^-F$ and $E(FAE)^-F$. In this paper, we derive a variety of properties of these constrained *g*-inverses by making use of the matrix rank method. As applications, we give some results on *g*-inverses of block matrices, and weighted least-squares estimators for the general linear model.

Keywords Linear matrix expression · Moore–Penrose inverse · Constrained generalized inverses · Matrix equation · Projector · Idempotent matrix · Rank equalities · General linear model · Weighted least-squares estimator