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# Joint distributions of numbers of runs of specified lengths in a sequence of Markov dependent multistate trials

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**Abstract** Let  $Z_0, Z_1, \dots, Z_n$  be a sequence of Markov dependent trials with state space  $\Omega = \{F_1, \dots, F_\lambda, S_1, \dots, S_\nu\}$ , where we regard  $F_1, \dots, F_\lambda$  as failures and  $S_1, \dots, S_\nu$  as successes. In this paper, we study the joint distribution of the numbers of  $S_i$ -runs of lengths  $k_{ij}$  ( $i = 1, 2, \dots, \nu, j = 1, 2, \dots, r_i$ ) based on four different enumeration schemes. We present formulae for the evaluation of the probability generating functions and the higher order moments of this distribution. In addition, when the underlying sequence is i.i.d. trials, the conditional distribution of the same run statistics, given the numbers of success and failure is investigated. We give further insights into the multivariate run-related problems arising from a sequence of the multistate trials. Besides, our results have potential applications to problems of various research areas and will come to prominence in the future.

**Keywords** Markov chain · Multistate trials · Runs · Moments · Enumeration schemes · Recursive scheme · Conditional distribution · Probability function · Probability generating function · Double generating function