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Integral representations and approximations for multivariate gamma distributions

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Abstract Let R be a $p \times p$-correlation matrix with an "m-factorial" inverse $R^{-1} = D - BB'$ with diagonal D minimizing the rank m of B. A new $\binom{m+1}{2}$-variate integral representation is given for p-variate gamma distributions belonging to R, which is based on the above decomposition of R^{-1} without the restriction $D > 0$ required in former formulas. This extends the applicability of formulas with small m. For example, every p-variate gamma cdf can be computed by an at most $\binom{p-1}{2}$-variate integral if $p = 3$ or $p = 4$. Since computation is only feasible for small m, a given R is approximated by an m-factorial R_0. The cdf belonging to R is approximated by the cdf associated with R_0 and some additional correction terms with the deviations between R and R_0.

Keywords Multivariate gamma distribution · Multivariate chi-square distribution · Multivariate Rayleigh-distribution · Approximation for positive definite matrices · m-factorial matrices