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Integral representations and approximations for multivariate gamma distributions

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Abstract Let $R$ be a $p \times p$-correlation matrix with an “$m$-factorial” inverse $R^{-1} = D - BB'$ with diagonal $D$ minimizing the rank $m$ of $B$. A new $(m + 1)/2$-variate integral representation is given for $p$-variate gamma distributions belonging to $R$, which is based on the above decomposition of $R^{-1}$ without the restriction $D > 0$ required in former formulas. This extends the applicability of formulas with small $m$. For example, every $p$-variate gamma cdf can be computed by an at most $(p-1)/2$-variate integral if $p = 3$ or $p = 4$. Since computation is only feasible for small $m$, a given $R$ is approximated by an $m$-factorial $R_0$. The cdf belonging to $R$ is approximated by the cdf associated with $R_0$ and some additional correction terms with the deviations between $R$ and $R_0$.

Keywords Multivariate gamma distribution · Multivariate chi-square distribution · Multivariate Rayleigh-distribution · Approximation for positive definite matrices · $m$-factorial matrices

1 Introduction and notations

For any $p \times p$-matrix $A = (a_{ij})$ the determinant is denoted by $|A|$ and the trace by $\text{tr}(A)$, $A > 0$ means positive definiteness, and $(a^{ij}) = A^{-1}$. $I_p$ or $I$ is a unit matrix and $E$ denotes the expectation of a random variable (r.v.). A cumulative distribution function is abbreviated by cdf and a probability density by pdf. Formulas from the handbook of mathematical functions by Abramowitz and Stegun (1965) are cited by “A.S” and their number.