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Second-order linearity of Wilcoxon statistics

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Abstract The rank statistic $S_n(\mathbf{t}) = 1/n \sum_{i=1}^{n} c_i R_i(\mathbf{t})$ ($\mathbf{t} \in \mathbb{R}^p$), with $R_i(\mathbf{t})$ being the rank of $e_i - \mathbf{t}^{\mathsf{T}}\mathbf{x}_i$, $i = 1, \ldots, n$ and $e_1, \ldots, e_n$ being the random sample from a distribution with a cdf F, is considered as a random process with $\mathbf{t}$ in the role of parameter. Under some assumptions on c_i, $\mathbf{x}_i$ and on the underlying distribution, it is proved that the process $\{S_n(\frac{\mathbf{t}}{\sqrt{n}}) - S_n(\mathbf{0}) - \mathsf{E}\, S_n(\mathbf{t}), |\mathbf{t}|_2 \leq M\}$ converges weakly to the Gaussian process. This generalizes the existing results where the one-dimensional case $\mathbf{t} \in \mathbb{R}$ was considered. We believe our method of the proof can be easily modified for the signed-rank statistics of Wilcoxon type. Finally, we use our results to find the second order asymptotic distribution of the R-estimator based on the Wilcoxon scores and also to investigate the length of the confidence interval for a single parameter β_l.

Keywords Rank statistics · Asymptotic linearity · Empirical processes · U-processes