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Statistical problems related to irrational rotations

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Abstract Let $\xi_i := \lfloor i\alpha + \beta \rfloor - \lfloor (i-1)\alpha + \beta \rfloor$ $(i = 1, 2, \ldots, m)$ be random variables as functions of β in the probability space $[0, 1)$ with the Lebesgue measure, where $\alpha \in [0, 1]$ is considered to be an unknown parameter which we want to estimate from the observation $\xi := \xi_1, \xi_2 \ldots \xi_m$. Let an observation ξ be given, which is a finite Sturmian sequence. We determine the likelihood function $P_\alpha(\xi)$ as a function of parameter α, and obtain the maximum likelihood estimator $\hat{\alpha}(\xi)$ as the relative frequency of 1s in a minimal cycle of ξ, where a factor η of ξ is called a minimal cycle if ξ is a factor of η^∞ and η has the minimum length among them. We also obtain a minimum sufficient statistics. The sample mean $(\xi_1 + \xi_2 + \cdots + \xi_m)/m$ which is an unbiased estimator of α is not admissible if $m = 6$ or $m \geq 8$ since it is not based on the minimum sufficient statistics.

Keywords Sturmian sequence · Irrational rotations · Minimum sufficient statistics · Admissible estimator · UMVUE