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Multiplicative correlations

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Abstract A multivariate distribution is said to have multiplicative correlation if the correlation matrix $R = (r_{ij})$ is written as $r_{ij} = \delta_i \delta_j$ or $r_{ij} = -\delta_i \delta_j (i \neq j)$ for a parameter vector $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_n)$. We first determine feasible values for $\boldsymbol{\delta}$ and show that variables with such a correlation matrix can always be decomposed into a common "signal" variable plus individual "noise" variables. It is also shown that a special case of this correlation matrix implies a sum constraint among variables and vice versa. Such properties illustrate why many multivariate distributions have such a correlation structure. Furthermore, several invariance properties lead to simple relations among several multivariate distributions.

Keywords Correlation modeling · Factorization of variables · Neural science · Partial correlation · Reduction method