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Estimation of a location parameter with restrictions or "vague information" for spherically symmetric distributions

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Abstract In this article we consider estimating a location parameter of a spherically symmetric distribution under restrictions on the parameter. First we consider a general theory for estimation on polyhedral cones which includes examples such as ordered parameters and general linear inequality restrictions. Next, we extend the theory to cones with piecewise smooth boundaries. Finally we consider shrinkage toward a closed convex set K where one has vague prior information that θ is in K but where θ is not restricted to be in K. In this latter case we give estimators which improve on the usual unbiased estimator while in the restricted parameter case we give estimators which improve on the projection onto the cone of the unbiased estimator. The class of estimators is somewhat non-standard as the nature of the constraint set may preclude weakly differentiable shrinkage functions. The technique of proof is novel in the sense that we first deduce the improvement results for the normal location problem and then extend them to the general spherically symmetric case by combining arguments about uniform distributions on the spheres, conditioning and completeness.

Keywords Convex cones · Integration-by-parts · Minimax estimate · Multivariate normal mean · Polyhedral cones · Positively homogeneous set · Quadratic loss · Spherically symmetric distribution · Weakly differentiable function