

## THE EMPIRICAL DISTRIBUTION FUNCTION AND PARTIAL SUM PROCESS OF RESIDUALS FROM A STATIONARY ARCH WITH DRIFT PROCESS

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**Abstract.** The weak convergence of the empirical process and partial sum process of the residuals from a stationary ARCH-M model is studied. It is obtained for any  $\sqrt{n}$  consistent estimate of the ARCH-M parameters. We find that the limiting Gaussian processes are no longer distribution free and hence residuals cannot be treated as i.i.d. In fact the limiting Gaussian process for the empirical process is a standard Brownian bridge plus an additional term, while the one for partial sum process is a standard Brownian motion plus an additional term. In the special case of a standard ARCH process, that is an ARCH process with no drift, the additional term disappears. We also study a sub-sampling technique which yields the limiting Gaussian processes for the empirical process and partial sum process as a standard Brownian bridge and a standard Brownian motion respectively.

*Key words and phrases:* Weak convergence, residuals, ARCH, drift, empirical distribution.