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NON-STANDARD ASYMPTOTICS IN AN INHOMOGENEOUS GAMMA PROCESS

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Abstract. Nonhomogeneous Poisson process (NHPP) is a commonly used stochastic model that is utilized to describe the pattern of repeated occurrence of certain events or conditions. An *inhomogeneous gamma process* evolves as a generalization to NHPP, where the observed failure epochs correspond to every successive κ-th event of the underlying Poisson process, κ being an unknown parameter to be estimated from the data. This article focuses on a special class of inhomogeneous gamma process, called *modulated power law process* (MPLP) that assumes the Weibull form of the intensity function. The traditional power law process is a popular stochastic formulation of certain empirical relationships between the time to failure and the cumulative number of failures, often observed in industrial experiments. The MPLP retains this underlying physical basis and provides a more flexible modeling environment potentially leading to a better fit to the failure data at hand. In this paper, we investigate inference issues related to MPLP. The maximum likelihood estimators (MLE's) of the model parameters are not in closed form and enjoy the curious property that they are asymptotically normal with a singular variance-covariance matrix. Consequently, the derivation of the large-sample results requires non-standard modifications of the usual arguments. We also propose a set of simple closed-form estimators that are asymptotically equivalent to the MLE's. Extensive simulation results are carried out to supplement the theoretical findings. Finally, we implement our inference results to a failure dataset arising from a repairable system.

Key words and phrases: Asymptotics, maximum likelihood estimation, modulated power law process, power law process, recurrent event.