RESAMPLING STUDENT’S $t$-TYPE STATISTICS

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Abstract. The present paper establishes conditional and unconditional central limit theorems for various resampling procedures for the $t$-statistic. The results work under fairly general conditions and the underlying random variables need not to be independent. Specific examples are then the $m(n)$ (double) bootstrap out of $k(n)$ observations, the Bayesian bootstrap and two-sample $t$-type permutation statistics. In case when $m(n)/k(n)$ is bounded away from zero and infinity necessary and sufficient conditions for the conditional central limit law of the bootstrap $t$-statistics are established. For high resampling intensity when $m(n)/k(n)$ tends to infinity the following general result is obtained. Without further other assumptions the bootstrap makes the resampled $t$-statistic automatically normal. The results are based on a general conditional limit theorem for weighted resampling statistics which is of own interest.

Key words and phrases: Student’s $t$-statistic, Welch statistic, two-sample permutation statistic, weighted bootstrap, double bootstrap, Bayesian bootstrap, central limit theorem, conditional central limit theorem.