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ESTIMATION OF THE MULTIVARIATE NORMAL PRECISION AND COVARIANCE MATRICES IN A STAR-SHAPE MODEL*

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Abstract. In this paper, we introduce the star-shape models, where the precision matrix $\mathbf{\Omega}$ (the inverse of the covariance matrix) is structured by the special conditional independence. We want to estimate the precision matrix under entropy loss and symmetric loss. We show that the maximal likelihood estimator (MLE) of the precision matrix is biased. Based on the MLE, an unbiased estimate is obtained. We consider a type of Cholesky decomposition of $\mathbf{\Omega}$, in the sense that $\mathbf{\Omega} = \mathbf{\Psi}'\mathbf{\Psi}$, where $\mathbf{\Psi}$ is a lower triangular matrix with positive diagonal elements. A special group $\mathcal{G}$, which is a subgroup of the group consisting all lower triangular matrices, is introduced. General forms of equivariant estimates of the covariance matrix and precision matrix are obtained. The invariant Haar measures on $\mathcal{G}$, the reference prior, and the Jeffreys prior of $\mathbf{\Psi}$ are also discussed. We also introduce a class of priors of $\mathbf{\Psi}$, which includes all the priors described above. The posterior properties are discussed and the closed forms of Bayesian estimators are derived under either the entropy loss or the symmetric loss. We also show that the best equivariant estimators with respect to $\mathcal{G}$ is the special case of Bayesian estimators. Consequently, the MLE of the precision matrix is inadmissible under either entropy or symmetric loss. The closed form of risks of equivariant estimators are obtained. Some numerical results are given for illustration.

Key words and phrases: Star-shape model, maximum likelihood estimator, precision matrix, covariance matrix, Jeffreys prior, reference prior, invariant Haar measure, Bayesian estimator, entropy loss, symmetric loss, inadmissibility.

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