

## JOINT DISTRIBUTIONS OF NUMBERS OF SUCCESS RUNS OF SPECIFIED LENGTHS IN LINEAR AND CIRCULAR SEQUENCES\*

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**Abstract.** In this paper, we study two joint distributions of the numbers of success runs of several lengths in a sequence of  $n$  Bernoulli trials arranged on a line (linear sequence) or on a circle (circular sequence) based on four different enumeration schemes. We present formulae for the evaluation of the joint probability functions, the joint probability generating functions and the higher order moments of these distributions. Besides, the present work throws light on the relation between the joint distributions of the numbers of success runs in the circular and linear binomial model. We give further insights into the run-related problems arisen from the circular sequence. Some examples are given in order to illustrate our theoretical results. Our results have potential applications to other problems such as statistical run tests for randomness and reliability theory.

*Key words and phrases:* Bernoulli trials, circular success runs, enumeration schemes, recursive scheme, circular binomial distribution of order  $k$ , probability function, probability generating function, double generating function.

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