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# VARIANCE ESTIMATION FOR SAMPLE QUANTILES USING THE $m$ OUT OF $n$ BOOTSTRAP

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**Abstract.** We consider the problem of estimating the variance of a sample quantile calculated from a random sample of size $n$. The $r$-th-order kernel-smoothed bootstrap estimator is known to yield an impressively small relative error of order $O(n^{-r/(2r+1)})$. It nevertheless requires strong smoothness conditions on the underlying density function, and has a performance very sensitive to the precise choice of the bandwidth. The unsmoothed bootstrap has a poorer relative error of order $O(n^{-1/4})$, but works for less smooth density functions. We investigate a modified form of the bootstrap, known as the $m$ out of $n$ bootstrap, and show that it yields a relative error of order smaller than $O(n^{-1/4})$ under the same smoothness conditions required by the conventional unsmoothed bootstrap on the density function, provided that the bootstrap sample size $m$ is of an appropriate order. The estimator permits exact, simulation-free, computation and has accuracy fairly insensitive to the precise choice of $m$. A simulation study is reported to provide empirical comparison of the various methods.

*Key words and phrases*: $m$ out of $n$ bootstrap, quantile, smoothed bootstrap.

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