

A GENERALIZED PÓLYA URN MODEL AND RELATED MULTIVARIATE DISTRIBUTIONS*

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Abstract. In this paper, we study a Pólya urn model containing balls of $(m + 1)$ different labels under a general replacement scheme, which is characterized by an $(m + 1) \times (m + 1)$ addition matrix of integers without constraints on the values of these $(m + 1)^2$ integers other than non-negativity. Let X_1, X_2, \dots, X_n be trials obtained by the Pólya urn scheme (with possible outcomes: “0”, “1”, \dots , “ m ”). We consider the multivariate distributions of the numbers of occurrences of runs of different types arising from the various enumeration schemes and give a recursive formula of the probability generating function. Some closed form expressions are derived as special cases, which have potential applications to various areas. Our methods for the derivation of the multivariate run-related distribution are very simple and suitable for numerical and symbolic calculations by means of computer algebra systems. The results presented here develop a general workable framework for the study of Pólya urn models. Our attempts are very useful for understanding non-classic urn models. Finally, numerical examples are also given in order to illustrate the feasibility of our results.

Key words and phrases: Pólya urn, replacement scheme, addition matrix, run, enumeration schemes, recursive scheme, probability generating function, double generating function, random structures.

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