

## TENSORS AND LIKELIHOOD EXPANSIONS IN THE PRESENCE OF NUISANCE PARAMETERS

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**Abstract.** Stochastic expansions of likelihood quantities are usually derived through ordinary Taylor expansions, rearranging terms according to their asymptotic order. The most convenient form for such expansions involves the score function, the expected information, higher order log-likelihood derivatives and their expectations. Expansions of this form are called expected/observed. If the quantity expanded is invariant or, more generally, a tensor under reparameterisations, the entire contribution of a given asymptotic order to the expected/observed expansion will follow the same transformation law. When there are no nuisance parameters, explicit representations through appropriate tensors are available. In this paper, we analyse the geometric structure of expected/observed likelihood expansions when nuisance parameters are present. We outline the derivation of likelihood quantities which behave as tensors under interest-respecting reparameterisations. This allows us to write the usual stochastic expansions of profile likelihood quantities in an explicitly tensorial form.

*Key words and phrases:* Asymptotic expansion, higher order asymptotics, interest respecting reparameterisation, nuisance parameter, profile likelihood, tensor.