

## ASYMPTOTIC DISTRIBUTIONS OF M-ESTIMATORS IN A SPATIAL REGRESSION MODEL UNDER SOME FIXED AND STOCHASTIC SPATIAL SAMPLING DESIGNS\*

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**Abstract.** In this paper, we consider M-estimators of the regression parameter in a spatial multiple linear regression model. We establish consistency and asymptotic normality of the M-estimators when the data-sites are generated by a class of deterministic as well as a class of stochastic spatial sampling schemes. Under the deterministic sampling schemes, the data-sites are located on a regular grid but may have an *infill* component. On the other hand, under the stochastic sampling schemes, locations of the data-sites are given by the realizations of a collection of independent random vectors and thus, are irregularly spaced. It is shown that scaling constants of different orders are needed for asymptotic normality under different spatial sampling schemes considered here. Further, in the stochastic case, the asymptotic covariance matrix is shown to depend on the spatial sampling density associated with the stochastic design. Results are established for M-estimators corresponding to certain non-smooth score functions including Huber's  $\psi$ -function and the sign functions (corresponding to the sample quantiles).

*Key words and phrases:* Central limit theorem, infill sampling, increasing-domain asymptotics, long range dependence, random field, strong mixing, stochastic design, spatial sampling design.

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