

## BIAS OF ESTIMATOR OF CHANGE POINT DETECTED BY A CUSUM PROCEDURE

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**Abstract.** For independent observations from a standard one-parameter exponential family, the estimator of change point after being detected by a CUSUM procedure is defined as the last zero point of the CUSUM process before the alarm time. By assuming that the change occurs far away from beginning and the control limit is large, an explicit form for the bias of estimator is derived conditioning on the change being detected. By further assuming that the change magnitude and its reference value approach zero at the same order, the local second order expansion of the bias is obtained for numerical evaluation. It is found that, surprisingly, even in the normal distribution case, the bias is non-zero when the change magnitude equals to its reference value, in contrast to the continuous time analog and the fixed sample size case. Numerical results show that the approximations are quite satisfactory.

*Key words and phrases:* Change-point estimator, CUSUM procedure, quasi-stationary bias, random walk theory, strong renewal theorem, ladder epoches and ladder heights.