

ON BAYES AND UNBIASED ESTIMATORS OF LOSS

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Abstract. We consider estimation of loss for generalized Bayes or pseudo-Bayes estimators of a multivariate normal mean vector, θ . In 3 and higher dimensions, the MLE X is UMVUE and minimax but is inadmissible. It is dominated by the James-Stein estimator and by many others. Johnstone (1988, On inadmissibility of some unbiased estimates of loss, *Statistical Decision Theory and Related Topics, IV* (eds. S. S. Gupta and J. O. Berger), Vol. 1, 361–379, Springer, New York) considered the estimation of loss for the usual estimator X and the James-Stein estimator. He found improvements over the Stein unbiased estimator of risk. In this paper, for a generalized Bayes point estimator of θ , we compare generalized Bayes estimators to unbiased estimators of loss. We find, somewhat surprisingly, that the unbiased estimator often dominates the corresponding generalized Bayes estimator of loss for priors which give minimax estimators in the original point estimation problem. In particular, we give a class of priors for which the generalized Bayes estimator of θ is admissible and minimax but for which the unbiased estimator of loss dominates the generalized Bayes estimator of loss. We also give a general inadmissibility result for a generalized Bayes estimator of loss.

Key words and phrases: Loss estimation, shrinkage estimation, Bayes estimation, unbiased estimation, superharmonicity.

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