Ann. Inst. Statist. Math.
Vol. 55, No. 3, 507–522 (2003)

ON MULTIVARIATE GAUSSIAN TAILS

ENKELEJD HASHORVA AND JÜRG HÜSLER

Department of Mathematical Statistics and Actuarial Sciences, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland, e-mail: enkelejd.hashorva@Allianz-Suisse.ch; juerg.huesler@stat.unibe.ch

(Received February 15, 2001; revised October 7, 2002)

Abstract. Let $\{\boldsymbol{X}_n, n \geq 1\}$ be a sequence of standard Gaussian random vectors in $\mathbb{R}^d$, $d \geq 2$. In this paper we derive lower and upper bounds for the tail probability $\boldsymbol{P}\{\boldsymbol{X}_n > \boldsymbol{t}_n\}$ with $\boldsymbol{t}_n \in \mathbb{R}^d$ some threshold. We improve for instance bounds on Mills ratio obtained by Savage (1962, *J. Res. Nat. Bur. Standards Sect. B*, **66**, 93–96). Furthermore, we prove exact asymptotics under fairly general conditions on both $\boldsymbol{X}_n$ and $\boldsymbol{t}_n$, as $\|\boldsymbol{t}_n\| \to \infty$ where the correlation matrix Σ_n of $\boldsymbol{X}_n$ may also depend on n.

Key words and phrases: Multivariate Mills ratio, Gaussian random sequences, tail asymptotics, quadratic programming.