CHARACTERIZATION OF THE LEAST CONCAVE MAJORANT OF BROWNIAN MOTION, CONDITIONAL ON A VERTEX POINT, WITH APPLICATION TO CONSTRUCTION

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Abstract. The characterization of the least concave majorant of Brownian motion by Pitman (1983, *Seminar on Stochastic Processes*, *1982* (eds. E. Cinlar, K. L. Chung and R. K. Getoor), 219–228, Birkhäuser, Boston) is tweaked, conditional on a vertex point. The joint distribution of this vertex point is derived and is shown to be generated with extreme ease. A procedure is then outlined by which one can construct the least concave majorant of a standard Brownian motion path over any finite, closed subinterval of $(0, \infty)$. This construction is exact in distribution. One can also construct a linearly interpolated version of the Brownian motion path (i.e. we construct the Brownian motion path over a grid of points and linearly interpolate) corresponding to this least concave majorant over the same finite interval. A discussion of how to translate the aforementioned construction to the least concave majorant of a Brownian bridge is also presented.

Key words and phrases: Brownian motion, least concave majorant.