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# ON A FAMILY OF DISTRIBUTIONS ATTAINING THE BHATTACHARYYA BOUND

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**Abstract.** A family of distributions for which an unbiased estimator of a function $g(\theta)$ of a real parameter $\theta$ can attain the second order Bhattacharyya lower bound is derived. Indeed, we obtain a necessary and sufficient condition for the attainment of the second order Bhattacharyya bound for a family of mixtures of distributions which belong to the exponential family. Furthermore, we give an example which does not satisfy this condition, but where the Bhattacharyya bound is attainable for a non-exponential family of distributions.

*Key words and phrases*: Cramér-Rao bound, exponential family, normal mixture, Bessel differential equation.