## STRONG CONSISTENCY OF AUTOMATIC KERNEL REGRESSION ESTIMATES\*

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Abstract. Regression function estimation from independent and identically distributed bounded data is considered. The  $L_2$  error with integration with respect to the design measure is used as an error criterion. It is shown that the kernel regression estimate with an arbitrary random bandwidth is weakly and strongly consistent for *all* distributions whenever the random bandwidth is chosen from some deterministic interval whose upper and lower bounds satisfy the usual conditions used to prove consistency of the kernel estimate for deterministic bandwidths. Choosing discrete bandwidths by cross-validation allows to weaken the conditions on the bandwidths.

*Key words and phrases*: Automatic kernel estimates, cross-validation, regression estimates, strong consistency.

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