GENERALIZED BINOMIAL AND NEGATIVE BINOMIAL DISTRIBUTIONS OF ORDER k BY THE ℓ-OVERLAPPING ENUMERATION SCHEME

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Abstract. In this paper, we investigate the exact distribution of the waiting time for the r-th ℓ-overlapping occurrence of success-runs of a specified length in a sequence of two state Markov dependent trials. The probability generating functions are derived explicitly, and as asymptotic results, relationships of a negative binomial distribution of order k and an extended Poisson distribution of order k are discussed. We provide further insights into the run-related problems from the viewpoint of the ℓ-overlapping enumeration scheme. We also study the exact distribution of the number of ℓ-overlapping occurrences of success-runs in a fixed number of trials and derive the probability generating functions. The present work extends several properties of distributions of order k and leads us a new type of geneses of the discrete distributions.

Key words and phrases: Run, waiting time, binomial distribution, negative binomial distribution, Poisson distribution, double generating function, probability generating function, Markov chain, Markov chain imbedding method.

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