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DENSITY ESTIMATION FOR A CLASS OF STATIONARY NONLINEAR PROCESSES

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Abstract. Let $\{X_t; t \in \mathbb{Z}\}$ be a strictly stationary nonlinear process of the form $X_t = \varepsilon_t + \sum_{r=1}^{\infty} W_{rt}$, where W_{rt} can be written as a function $g_r(\varepsilon_{t-1}, \ldots, \varepsilon_{t-r-q})$, $\{\varepsilon_t; t \in \mathbb{Z}\}$ is a sequence of independent and identically distributed (*i.i.d.*) random variables with $E|\varepsilon_1|^g < \infty$ for some $\gamma > 0$ and $q \geq 0$ is a fixed integer. Under certain mild regularity conditions on g_r and $\{\varepsilon_t\}$ we then show that X_1 has a density function f and that the standard kernel type estimator $\widehat{f}_n(x)$ based on a realization $\{X_1, \ldots, X_n\}$ from $\{X_t\}$ is, asymptotically, normal and converges a.s. to $f(x)$ as $n \to \infty$.

Key words and phrases: Nonlinear process, kernel type density estimators, bilinear process, central limit theorem, almost sure convergence.

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