LINEAR VERSUS NONLINEAR RULES FOR MIXTURE NORMAL PRIORS

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Abstract. The problem under consideration is the Γ-minimax estimation, under L_2 loss, of a multivariate normal mean when the covariance matrix is known. The family Γ of priors is induced by mixing zero mean multivariate normals with covariance matrix τI by nonnegative random variables τ, whose distributions belong to a suitable family $\mathcal{G}$. For a fixed family $\mathcal{G}$, the linear (affine) Γ-minimax rule is compared with the usual Γ-minimax rule in terms of corresponding Γ-minimax risks. It is shown that the linear rule is "good", i.e., the ratio of the risks is close to 1, irrespective of the dimension of the model. We also generalize the above model to the case of nonidentity covariance matrices and show that independence of the dimensionality is lost in this case. Several examples illustrate the behavior of the linear Γ-minimax rule.

Key words and phrases: Affine rules, efficiency, Bayes risk, Brown's identity, Γ-minimax rules.