CONSERVED QUANTITIES AND SYMMETRIES RELATED TO STOCHASTIC DYNAMICAL SYSTEMS

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Abstract. The present article focuses on the three topics related to the notions of "conserved quantities" and "symmetries" in stochastic dynamical systems described by stochastic differential equations of Stratonovich type. The first topic is concerned with the relation between conserved quantities and symmetries in stochastic Hamilton dynamical systems, which is established in a way analogous to that in the deterministic Hamilton dynamical theory. In contrast with this, the second topic is devoted to investigate the procedures to derive conserved quantities from symmetries of stochastic dynamical systems without using either the Lagrangian or Hamiltonian structure. The results in these topics indicate that the notion of symmetries is useful for finding conserved quantities in various stochastic dynamical systems. As a further important application of symmetries, the third topic treats the similarity method to stochastic dynamical systems. That is, it is shown that the order of a stochastic system can be reduced, if the system admits symmetries. In each topic, some illustrative examples for stochastic dynamical systems and their conserved quantities and symmetries are given.

Key words and phrases: Stochastic dynamical systems, conserved quantities, symmetries, similarity method.