ON A CHARACTERIZATION OF THE GAMMA DISTRIBUTION: THE INDEPENDENCE OF THE SAMPLE MEAN AND THE SAMPLE COEFFICIENT OF VARIATION

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Abstract. Let $n \geq 3$ and let $X_1, \dots, X_n$ be positive i.i.d. random variables whose common distribution function f has a continuous p.d.f. Using earlier work of the present authors and a method due to Anosov for solving certain integro-functional equations, it is shown that the independence of the sample mean and the sample coefficient of variation is equivalent to that f is a gamma function. While the proof is of methodological interest, this conclusion can also be arrived at without any assumptions by appealing to the Laplace-Stieltjes transform, as in the Concluding Remark (Section 3).

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