NONCONSERVATIVE ESTIMATING FUNCTIONS AND APPROXIMATE QUASI-LIKELIHOODS

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Abstract. The estimating function approach unifies two dominant methodologies in statistical inferences: Gauss's least square and Fisher's maximum likelihood. However, a parallel likelihood inference is lacking because estimating functions are in general not integrable, or nonconservative. In this paper, nonconservative estimating functions are studied from vector analysis perspective. We derive a generalized version of the Helmholtz decomposition theorem for estimating functions of any dimension. Based on this theorem we propose locally quadratic potentials as approximate quasi-likelihoods. Quasi-likelihood ratio tests are studied. The ideas are illustrated by two examples: (a) logistic regression with measurement error model and (b) probability estimation conditional on marginal frequencies.

Key words and phrases: Divergence-free vector fields, generalized Helmholtz decomposition, gradient vector fields, logistic regression with measurement error, potentials, quasi-likelihood ratio test, quasi-scores.