ON TYLER'S M-FUNCTIONAL OF SCATTER IN HIGH DIMENSION

LUTZ DÜMBGEN

Institut für Angewandte Mathematik, Universität Heidelberg, Im Neuenheimer Feld 294
D-69120 Heidelberg, Germany

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Abstract. Let $y_1, y_2, \ldots, y_n \in \boldsymbol{R}^q$ be independent, identically distributed random vectors with nonsingular covariance matrix Σ, and let $S = S(y_1, \ldots, y_n)$ be an estimator for Σ. A quantity of particular interest is the condition number of $\Sigma^{-1}S$. If the y_i are Gaussian and S is the sample covariance matrix, the condition number of $\Sigma^{-1}S$, i.e. the ratio of its extreme eigenvalues, equals $1 + O_{\mathrm{p}}((q/n)^{1/2})$ as $q \to \infty$ and $q/n \to 0$. The present paper shows that the same result can be achieved with two estimators based on Tyler's (1987, *Ann. Statist.*, **15**, 234–251) M-functional of scatter, assuming only elliptical symmetry of $\mathcal{L}(y_i)$ or less. The main tool is a linear expansion for this M-functional which holds uniformly in the dimension q. As a by-product we obtain continuous Fréchet-differentiability with respect to weak convergence.

Key words and phrases: Differentiability, dimensional asymptotics, elliptical symmetry, M-functional, scatter matrix, symmetrization.