INFERENCE FOR THE TAIL PARAMETERS OF A LINEAR PROCESS WITH HEAVY TAIL INNOVATIONS

SOMNATH DATTA AND WILLIAM P. MCCORMICK

Department of Statistics, Franklin College of Arts and Sciences, University of Georgia, 204 Statistics Building, Athens, GA 30602-1952, U.S.A.

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Abstract. Consider a linear process $X_t = \sum_{i=0}^{\infty} c_i Z_{t-i}$, where the innovations Z's are i.i.d. satisfying a standard tail regularity and balance condition, viz., $P(Z > z) \sim rz^{-\alpha}L_1(z)$, $P(Z < -z) \sim sz^{-\alpha}L_1(z)$, as $z \to \infty$, where $r + s = 1$, $r, s \geq 0$, $\alpha > 0$ and L_1 is a slowly varying function. It turns out that in this setup, $P(X > x) \sim px^{-\alpha}L(x)$, $P(X < -x) \sim qx^{-\alpha}L(x)$, as $x \to \infty$, where α is the same as above, p is a convex combination of r and s, $p + q = 1$, $p, q \geq 0$ and $L = \|\underline{c}\|_\alpha^\alpha L_1$ where $\|\underline{c}\|_\alpha = (\Sigma|c_i|^\alpha)^{1/\alpha}$. The quantities α and $\beta = 2p - 1$ can be regarded as tail parameters of the marginal distribution of X_t. We estimate α and β based on a finite realization $X_1, \ldots, X_n$ of the time series. Consistency and asymptotic normality of the estimators are established. As a further application, we estimate a tail probability under the marginal distribution of the X_t. A small simulation study is included to indicate the finite sample behavior of the estimators.

Key words and phrases: Linear processes, heavy tailed distribution, tail parameters, tail probability.