JOINT DISTRIBUTIONS OF NUMBERS OF SUCCESS-RUNS UNTIL THE FIRST CONSECUTIVE k SUCCESSES IN A HIGHER-ORDER TWO-STATE MARKOV CHAIN

MASAYUKI UCHIDA

The Institute of Statistical Mathematics, 4-6-7 Minami-Azabu, Minato-ku, Tokyo 106-8569, Japan

(Received March 13, 1996; revised May 8, 1997)

Abstract. Let $X_{-m+1}, X_{-m+2}, \ldots, X_0, X_1, X_2, \ldots$ be a time-homogeneous $\{0,1\}$-valued m-th order Markov chain. Joint distributions of the numbers of trials, failures and successes, of the numbers of trials and success-runs of length l ($m \leq l \leq k$) and of the numbers of trials and success-runs of length l ($l \leq m \leq k$) until the first consecutive k successes are obtained in the sequence $X_1, X_2, \ldots$. There are some ways of counting numbers of runs of length l. This paper studies the joint distributions based on four ways of counting numbers of runs, i.e., the number of non-overlapping runs of length l, the number of runs of length greater than or equal to l, the number of overlapping runs of length l and the number of runs of length exactly l. Marginal distributions of them can be obtained immediately, and surprisingly their distributions are very simple.

Key words and phrases: Probability generating function, discrete distribution, success and failure runs, geometric distribution, geometric distribution of order k, higher order Markov chain.