LOCAL POLYNOMIAL REGRESSION: OPTIMAL KERNELS AND ASYMPTOTIC MINIMAX EFFICIENCY*

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Abstract. We consider local polynomial fitting for estimating a regression function and its derivatives nonparametrically. This method possesses many nice features, among which automatic adaptation to the boundary and adaptation to various designs. A first contribution of this paper is the derivation of an optimal kernel for local polynomial regression, revealing that there is a universal optimal weighting scheme. Fan (1993, *Ann. Statist.*, **21**, 196–216) showed that the univariate local linear regression estimator is the best linear smoother, meaning that it attains the asymptotic linear minimax risk. Moreover, this smoother has high minimax risk. We show that this property also holds for the multivariate local linear regression estimator. In the univariate case we investigate minimax efficiency of local polynomial regression estimators, and find that the asymptotic minimax efficiency for commonly-used orders of fit is 100% among the class of all linear smoothers. Further, we quantify the loss in efficiency when going beyond this class.

Key words and phrases: Curve estimation, local polynomials, minimax efficiency, minimax risk, multivariate curve estimation, nonparametric regression, universal optimal weighting scheme.