QUANTILE PROCESSES IN THE PRESENCE OF AUXILIARY INFORMATION

BIAO ZHANG

Department of Mathematics, The University of Toledo, Toledo, OH 43606, U.S.A.

(Received March 22, 1995; revised April 24, 1996)

Abstract. We employ the empirical likelihood method to propose a modified quantile process under a nonparametric model in which we have some auxiliary information about the population distribution. Furthermore, we propose a modified bootstrap method for estimating the sampling distribution of the modified quantile process. To explore the asymptotic behavior of the modified quantile process and to justify the bootstrapping of this process, we establish the weak convergence of the modified quantile process to a Gaussian process and the almost-sure weak convergence of the modified bootstrapped quantile process to the same Gaussian process. These results are demonstrated to be applicable, in the presence of auxiliary information, to the construction of asymptotic bootstrap confidence bands for the quantile function. Moreover, we consider estimating the population semi-interquartile range on the basis of the modified quantile process. Results from a simulation study assessing the finite-sample performance of the proposed semi-interquartile range estimator are included.

Key words and phrases: Bootstrap, Brownian bridge, confidence band, empirical likelihood, empirical process, Gaussian process, semi-interquartile range, weak convergence.