DIFFERENTIABLE FUNCTIONALS AND SMOOTHED BOOTSTRAP

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Abstract. The differentiability properties of statistical functionals have several interesting applications. We are concerned with two of them. First, we prove a result on asymptotic validity for the so-called *smoothed bootstrap* (where the artificial samples are drawn from a density estimator instead of being resampled from the original data). Our result can be considered as a smoothed analog of that obtained by Parr (1985, *Stat. Probab. Lett.*, **3**, 97–100) for the standard, unsmoothed bootstrap. Second, we establish a result on asymptotic normality for estimators of type $T_n = T(\hat{f}_n)$ generated by a density functional $T = T(f)$, $\hat{f}_n$ being a density estimator. As an application, a quick and easy proof of the asymptotic normality of $\int \hat{f}_n^2$, (the *plug-in* estimator of the integrated squared density $\int f^2$) is given.

Key words and phrases: Smoothed bootstrap, differentiable statistical functionals, bootstrap validity, smoothed empirical process, integrated squared densities.