DIAGNOSING BOOTSTRAP SUCCESS

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(Received August 8, 1995; revised February 13, 1996)

Abstract. We show that convergence of intuitive bootstrap distributions to the correct limit distribution is equivalent to a local asymptotic equivariance property of estimators and to an asymptotic independence property in the bootstrap world. The first equivalence implies that bootstrap convergence fails at superefficiency points in the parameter space. However, superefficiency is only a sufficient condition for bootstrap failure. The second equivalence suggests graphical diagnostics for detecting whether or not the intuitive bootstrap is trustworthy in a given data analysis. Both criteria for bootstrap convergence are related to Hájek's (1970, *Zeit. Wahrscheinlichkeitsth.*, **14**, 323–330) formulation of the convolution theorem and to Basu's (1955, *Sankhyā*, **15**, 377–380) theorem on the independence of an ancillary statistic and a complete sufficient statistic.

Key words and phrases: Bootstrap convergence, local asymptotic equivariance, local asymptotic sufficiency, asymptotic independence, superefficiency points, convolution theorem.