ON SEQUENTIAL FIXED-WIDTH CONFIDENCE INTERVALS FOR THE MEAN AND SECOND-ORDER EXPANSIONS OF THE ASSOCIATED COVERAGE PROBABILITIES

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Abstract. In order to construct fixed-width (2d) confidence intervals for the mean of an unknown distribution function F, a new purely sequential sampling strategy is proposed first. The approach is quite different from the more traditional methodology of Chow and Robbins (1965, *Ann. Math. Statist.*, **36**, 457–462). However, for this new procedure, the coverage probability is shown (Theorem 2.1) to be at least $(1-\alpha)+Ad^2+o(d^2)$ as $d \to 0$ where $(1-\alpha)$ is the preassigned level of confidence and A is an appropriate functional of F, under some regularity conditions on F. The rates of convergence of the coverage probability to $(1-\alpha)$ obtained by Csenki (1980, *Scand. Actuar. J.*, 107–111) and Mukhopadhyay (1981, *Comm. Statist. Theory Methods*, **10**, 2231–2244) were merely $O(d^{1/2-q})$, with $0 < q < 1/2$, under the Chow-Robbins stopping time τ^*. It is to be noted that such considerable sharpening of the rate of convergence of the coverage probability is achieved even though the new stopping variable is $O_P(\tau^*)$. An accelerated version of the stopping rule is also provided together with the analogous second-order characteristics. In the end, an example is given for the mean estimation problem of an exponential distribution.

Key words and phrases: Distribution-free, fixed-width confidence intervals, confidence level, second-order expansions, purely sequential, accelerated sequential, Markov inequality.