BAHADUR EFFICIENCY AND ROBUSTNESS OF STUDENTIZED SCORE TESTS

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Abstract. We derive the exact Bahadur slopes of studentized score tests for a simple null hypothesis in a one-parameter family of distributions. The Student's t-test is included as a special case for which a recent result of Rukhin (1993, *Sankhyā Ser. A*, **55**, 159–163) was improved upon. It is shown that locally optimal Bahadur efficiency for one-sample location models with a known or estimated scale parameter is attained within the class of studentized score tests. The studentized test has an asymptotic null distribution free of the scale parameter, and the optimality of likelihood scores does not depend on the existence of a moment generating function. We also consider the influence function and breakdown point of such tests as part of our robustness investigation. The influence of any studentized score test is bounded from above, indicating certain degree of robustness of validity, but a bounded score function is needed to cap the influence from below and to ensure a high power breakdown point. We find that the standard Huber-type score tests are not only locally minimax in Bahadur efficiency, but also very competitive in global efficiency at a variety of location models.

Key words and phrases: Bahadur slope, efficiency, influence function, score test.