MOMENT SOLUTION TO AN URN MODEL

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Abstract. A subset of Bernard's RD-model (replenishment-depletion) is considered from the viewpoint of the calculus of finite differences. The most general case is considered and includes an urn with balls of many colors, each color being replenished either deterministically or stochastically. Factorial moment generating functions (fmgfs) are employed to define probability generating functions. A new result is given for the two color case defining the fmgf and probability generating function (with probabilities) when the replenishments are positive valued random variables with given factorial moments. This result involves beta integral transforms defining a manifold of discrete distributions. Particular cases relate to hypergeometric discrete distributions.

Key words and phrases: Bernard's urn, beta integral transforms, finite difference calculus, generating function, hypergeometric distributions, hypergeometric functions, moments, replenishment-depletion urn.