LAN OF EXTREME ORDER STATISTICS

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Abstract. Consider an iid sample $Z_1, \ldots, Z_n$ with common distribution function F on the real line, whose upper tail belongs to a parametric family $\{F_\beta : \beta \in \Theta\}$. We establish local asymptotic normality (LAN) of the loglikelihood process pertaining to the vector $(Z_{n-i+1:n})_{i=1}^k$ of the upper $k = k(n) \to_{n\to\infty} \infty$ order statistics in the sample, if the family $\{F_\beta : \beta \in \Theta\}$ is in a neighborhood of the family of generalized Pareto distributions. It turns out that, except in one particular location case, the kth-largest order statistic $Z_{n-k+1:n}$ is the central sequence generating LAN. This implies that $Z_{n-k+1:n}$ is asymptotically sufficient and that asymptotically optimal tests for the underlying parameter β can be based on the single order statistic $Z_{n-k+1:n}$. The rate at which $Z_{n-k+1:n}$ becomes asymptotically sufficient is however quite poor.

Key words and phrases: Extreme order statistics, local asymptotic normality, central sequence, generalized Pareto distributions, asymptotic sufficiency, optimal tests.