ON CONSTRUCTION OF IMPROVED ESTIMATORS IN MULTIPLE-DESIGN MULTIVARIATE LINEAR MODELS UNDER GENERAL RESTRICTION

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Abstract. Consider a set of p equations $\boldsymbol{Y}_i = \boldsymbol{X}_i\boldsymbol{\xi}_i + \boldsymbol{\epsilon}_i$, $i = 1, \ldots, p$, where the rows of the random error matrix $(\boldsymbol{\epsilon}_1, \ldots, \boldsymbol{\epsilon}_p) : n \times p$ are mutually independent and identically distributed with a p-variate distribution function $F(\boldsymbol{x})$ having null mean and finite positive definite variance-covariance matrix $\boldsymbol{\Sigma}$. We are mainly interested in an improvement upon a feasible generalized least squares estimator (FGLSE) for $\boldsymbol{\xi} = (\boldsymbol{\xi}_1', \ldots, \boldsymbol{\xi}_p')'$ when it is a priori suspected that $\boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{c}_0$ may hold. For this problem, Saleh and Shiraishi (1992, *Nonparametric Statistics and Related Topics* (ed. A. K. Md. E. Saleh), 269–279, North-Holland, Amsterdam) investigated the property of estimators such as the shrinkage estimator (SE), the positive-rule shrinkage estimator (PSE) in the light of their asymptotic distributional risks associated with the Mahalanobis loss function. We consider a general form of estimators and give a sufficient condition for proposed estimators to improve on FGLSE with respect to their asymptotic distributional quadratic risks (ADQR). The relative merits of these estimators are studied in the light of the ADQR under local alternatives. It is shown that the SE, the PSE and the Kubokawa-type shrinkage estimator (KSE) outperform the FGLSE and that the PSE is the most effective among the four estimators considered under $\boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{c}_0$. It is also observed that the PSE and the KSE fairly improve over the FGLSE. Lastly, the construction of estimators improved on a generalized least squares estimator is studied, assuming normality when $\boldsymbol{\Sigma}$ is known.

Key words and phrases: Shrinkage estimators, generalized least squares estimators, asymptotic distribution, seemingly unrelated regression model.