JOINT DISTRIBUTIONS OF NUMBERS OF SUCCESS-RUNS AND FAILURES UNTIL THE FIRST CONSECUTIVE k SUCCESSES

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Abstract. Joint distributions of the numbers of failures, successes and success-runs of length less than k until the first consecutive k successes are obtained for some random sequences such as a sequence of independent and identically distributed integer valued random variables, a $\{0,1\}$-valued Markov chain and a binary sequence of order k. There are some ways of counting numbers of runs with a specified length. This paper studies the joint distributions based on three ways of counting numbers of runs, i.e., the number of overlapping runs with a specified length, the number of non-overlapping runs with a specified length and the number of runs with a specified length or more. Marginal distributions of them can be derived immediately, and most of them are surprisingly simple.

Key words and phrases: Probability generating function, geometric distribution, discrete distributions, Markov chain, waiting time, geometric distribution of order k, binary sequence of order k.