ON THE APPROXIMATION OF CONTINUOUS TIME THRESHOLD ARMA PROCESSES

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Abstract. Threshold autoregressive (AR) and autoregressive moving average (ARMA) processes with continuous time parameter have been discussed in several recent papers by Brockwell *et al.* (1991, *Statist. Sinica*, **1**, 401–410), Tong and Yeung (1991, *Statist. Sinica*, **1**, 411–430), Brockwell and Hyndman (1992, *International Journal Forecasting*, **8**, 157–173) and Brockwell (1994, *J. Statist. Plann. Inference*, **39**, 291–304). A threshold ARMA process with boundary width $2\delta > 0$ is easy to define in terms of the unique strong solution of a stochastic differential equation whose coefficients are piecewise linear and Lipschitz. The positive boundary-width is a convenient mathematical device to smooth out the coefficient changes at the boundary and hence to ensure the existence and uniqueness of the strong solution of the stochastic differential equation from which the process is derived. In this paper we give a direct definition of a threshold ARMA processes with $\delta = 0$ in the important case when only the autoregressive coefficients change with the level of the process. (This of course includes all threshold AR processes with constant scale parameter.) The idea is to express the distributions of the process in terms of the weak solution of a certain stochastic differential equation. It is shown that the joint distributions of this solution with $\delta = 0$ are the weak limits as $\delta \downarrow 0$ of the distributions of the solution with $\delta > 0$. The sense in which the approximating sequence of processes used by Brockwell and Hyndman (1992, *International Journal Forecasting*, **8**, 157–173) converges to this weak solution is also investigated. Some numerical examples illustrate the value of the latter approximation in comparison with the more direct representation of the process obtained from the Cameron-Martin-Girsanov formula. It is used in particular to fit continuous-time threshold models to the sunspot and Canadian lynx series.

Key words and phrases: Non-linear time series, continuous-time ARMA process, threshold model, stochastic differential equation, approximation of diffusion processes.