NECESSARY CONDITIONS FOR CHARACTERIZATION OF LAWS VIA MIXED SUMS

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Abstract. Suppose X and Y are independent and identically distributed, and independent of U which satisfies $0 \leq U \leq 1$. Recent work has centered on finding the laws $\mathcal{L}(X)$ for which $X \cong U(X+Y)$ where $\cong$ denotes equality in law. We show that this equation corresponds to a certain projective invariance property under random rotations. Implicitly or explicitly, it has been assumed that the characteristic function of X has an expansion property near the origin. We show that solutions may be admitted in the absence of this condition when $-\log U$ has a lattice law. A continuous version of the basic problem replaces sums with a Lévy process. Instead we consider self-similar processes, showing that a solution exists only when U is constant, and then all processes of a given order are admitted.

Key words and phrases: Distribution theory, characterization, semi-stable laws, mixtures, self-similar processes.