ESTIMATING NON-LINEAR FUNCTIONS OF THE SPECTRAL DENSITY, USING A DATA-TAPER

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Abstract. Let $f(\omega)$ be the spectral density of a Gaussian stationary process. Consider periodogram-based estimators of integrals of certain non-linear functions ζ of $f(\omega)$, like $H_T := \int_{-\pi}^{\pi} \Lambda(\omega)\zeta(I_T(\omega))d\omega$, where $\Lambda(\omega)$ is a bounded function of bounded variation, possibly depending on the sample size T. Then it is known that, under mild conditions on ζ, a central limit theorem holds for these statistics H_T if the non-tapered periodogram $I_T(\omega)$ is used. In particular, Taniguchi (1980, *J. Appl. Probab.*, **17**, 73–83) gave a consistent and asymptotic normal estimator of $\int_{-\pi}^{\pi} \Lambda(\omega)\Phi(f(\omega))d\omega$, choosing ζ to be a suitable transform of a given function Φ. In this work we shall generalize this result to statistics H_T where a taper-modified periodogram is used. We apply our result to the use of data-tapers in nonparametric peak-insensitive spectrum estimation. This was introduced in von Sachs (1994, *J. Time Ser. Anal.*, **15**, 429–452) where the performance of this estimator was shown to be substantially improved by using a taper.

Key words and phrases: Gaussian stationary process, spectral density, periodogram, data-taper, peak-insensitive spectral estimator.