LBI TESTS OF INDEPENDENCE IN BIVARIATE EXPONENTIAL DISTRIBUTIONS

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Abstract. The locally best invariant test for the hypothesis of independence in bivariate distributions with exponentially distributed marginals is derived. The model consists of a family of bivariate exponential distributions with probability density function

$$f_\theta(x_1, x_2; \lambda_1, \lambda_2) = \lambda_1 \lambda_2 \exp[-(\lambda_1 x_1 + \lambda_2 x_2)] g(\lambda_1 x_1, \lambda_2 x_2; \theta)$$

with unknown scale parameter λ_j $(j = 1, 2)$ and association parameter θ which includes the independence situation. The locally best invariant (LBI) test is derived and the asymptotic null and nonnull distributions are also derived under some regularity conditions. The results are applied to the Gumbel (1960, *J. Amer. Statist. Assoc.*, **55**, 698–707), Frank (1979, *Aequationes Math.*, **19**, 194–226), and Cook and Johnson (1981, *J. Roy. Statist. Soc. Ser. B*, **43**, 210–218) families.

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